MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

22 OCTOBER 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.
2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.
3. Write all your answers in the Answer Booklet provided.

Question 1 [10 marks]

a. Let f be given by

$$f(x)=\begin{cases} x^2-x+1, & \text{if } x\le 1 \\ kx^2+2, & \text{if } x>1 \end{cases}$$

For which value(s) of k is f continuous on its domain? (3 marks)

b. Find the following limits.

i. $\lim\limits_{x\to 4}\dfrac{x^2+x-20}{x-4}$ (2 marks)

ii. $\lim\limits_{s\to\infty}\dfrac{s^4+s^2+13}{s^3+8s+9}$ (2 marks)

iii. $\lim\limits_{t\to 0}\dfrac{t}{\sqrt{4-t}-2}$ (3 marks)

Question 2 [10 marks]

a. Find the equation of the tangent line at $(0,3)$ to the curve $y^2e^{2x}=3y+x^2$. (5 marks)

b. Use integration by parts to evaluate $\int(3t-t^2)\sin(2t)\,dt$ (5 marks)

Question 3 [10 marks]

a. Given $f(x)=5-8x^3-x^4$.

i. Identify the critical numbers and determine the intervals on which the function increases and decreases. Classify the critical points as local maximum, local minimum or neither. (3 marks)

ii. Determine the intervals on which the function is concave up and concave down and the x-coordinate of the inflection point(s). (3 marks)

Continued...

b. You want to construct a cylindrical can with a bottom but no top that will have a volume of 27 cm^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

(4 marks)

Question 4 [10 marks]

a. Sketch the curve $x = y^2 - y - 6$ and the line $x = 2y + 4$.

(1 mark)

i. Find the coordinates of the points of intersection of the line and the curve. (1 mark)

ii. Find the area of the region enclosed by the curve and the line.

(3 marks)

b. The region bounded by $y = 7 - x^2, x = -2, x = 2$ and the x-axis is rotated completely about the x-axis. Find the volume of the solid formed.

(5 marks)

Question 5 [10 marks]

a. Find the general solution for the separable differential equation $\frac{y^2}{x}\frac{dy}{dx} = 1 + x^2$.

(4 marks)

b. Find the unique solution for the second order differential equation

$$y'' + y' - 6y = 0 \text{ where } y(0) = 4 \text{ and } y'(0) = -3$$

(6 marks)

End of Page

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)dx = c\int f(x)dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$